



White Paper

Title: Lambda Calculation – The Modified Brettschneider Equation – Corrected for Gasoline NDIR Measurement

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The Brettschneider equation is the de-facto standard method used to calculate the normalized air/fuel balance (Lambda) for domestic and international I&M inspection programs. It is taken from a paper written by Dr. Johannes Brettschneider, at Robert Bosch in 1979 and published in “Bosch technische Berichte”, Vol 6 (1979) N0. 4, Pgs 177-186. In the paper, Dr. Brettschneider established a method to calculate Lambda (Balance of Oxygen to Fuel) by comparing the ratio of oxygen molecules to carbon and hydrogen molecules in the exhaust. It has been the standard equation used in NDIR exhaust gas measurement instruments since the early 1980’s.

Work done by Brett C. Singer and Robert A. Harley published in 1998 indicates an error in the conventionally accepted equation, in that Brettschneider assumed that (since gasoline vapors are measured as ‘equivalent hexane’ – he could simply multiply HC by 6 in order to account for the carbon atoms represented by % HC. Singer and Harley discovered through correlation of FTIR carbon measurement to NDIR HC measurement that conventional NDIR measurement techniques missed about 50% of the carbon in the exhaust gas HC mix, due to the low NDIR absorption of double-bond Carbon-bearing compounds. The Modified Brettschneider Equation below corrects for this error by multiplying the reported NDIR HC by a factor of 12 instead of 6.

Note that only Gasoline is effected by this modification, as gasoline vapors are a mix of hydrocarbon compounds, while Propane and Methane are not.

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$$\lambda = \frac{[CO_2] + \left[\frac{CO}{2}\right] + [O_2] + \left[\frac{NO}{2}\right] + \left(\left(\frac{H_{cv}}{4} \times \frac{3.5}{3.5 + \frac{[CO]}{[CO_2]}} \right) - \frac{O_{cv}}{2} \right) \times ([CO_2] + [CO])}{\left(1 + \frac{H_{cv}}{4} - \frac{O_{cv}}{2} \right) \times ([CO_2] + [CO] + (n \times [HC]))}$$

Where:

[XX] = Gas Concentration in % Volume.

(You have to convert PPM HC to % HC by dividing it by 10,000)

H_{cv} = Atomic ratio of Hydrogen to Carbon in the fuel.

O_{cv} = Atomic ratio of Oxygen to Carbon in the fuel.

n = Number of carbon atoms in a molecule of the selected HC.

n = 12 for Hexane (Gasoline), 3 for Propane (LPG), 1 for Methane (CNG)

The equation is a little complex, but is relatively easily calculated from the measured values of CO, CO₂, unburned HC, and unconsumed O₂ in the exhaust:

The equation above compares all of the oxygen in the numerator to the oxygen demand created by the combustibles in the denominator. (The equation has been modified to allow for Propane and Methane as fuels, and to compensate for the fact that ½ the carbon atoms in Gasoline exhaust are not measured by the NDIR method conventionally used for HC. The result of the calculation is the term ‘Lambda’ (λ) a dimensionless term that relates to the stoichiometric value of air to fuel. At the stoichiometric point, Lambda = 1.000. A Lambda value of 1.050 is about 5.0% lean, and a Lambda value of 0.950 is about 5.0% rich. Once Lambda is calculated, A/F ratio can be easily determined by simply multiplying Lambda times the stoichiometric A/F ratio for the fuel selected - e.g. 14.71 for gasoline, 15.87 for LPG, and 17.45 for CNG.

Details of the Brettschneider Equation:

Although this equation may be difficult to understand in theory, it is simple to use in practice. The equation directly reflects the ‘degree of lean-ness’ of the air/fuel mixture – and is largely independent how efficiently the fuel is oxidized – a very important factor to consider when dealing specifically with air / fuel balance issues. The manner in which

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this equation is to be used is strictly a function of the application though, and it is an excellent replacement for more commonly used conventions, such as CO measurement for rich-side applications (performance tuning), 'wide range lambda sensors', which are not only very non-linear, but also very sensitive to combustibles in the exhaust stream, or EGT, which is a combination of flame temperature and volume (power).

The only stable air/fuel ratio measurement that we have found to date is one that first makes an accurate measure of the constituent gases in the exhaust stream (at least the four gases of HC, CO, CO₂ and O₂) and calculates the oxygen and combustibles content and then the lambda and A/F value as above.

The Relationship between Lambda and A/F ratio:

Because Lambda = 1.000 when the oxygen and combustibles are in perfect stoichiometric balance, Lambda can easily be used to calculate A/F ratio for particular fuels.

The active A/F ratio is simply the calculated Lambda times the stoichiometric A/F ratio for the specific fuel used (14.71 for gasoline, but other fuels have different values)

$$\text{AFR} = (\text{Stoich AFR}) \times \text{Lambda}$$

Thus, if Stoich AFR = 14.71, and Lambda = 0.950, AFR = 13.97.

This method is far superior to other approaches which use only one gas (CO or Oxygen) to approximate A/F ratio – as the Brettschneider method uses all of the oxygen and carbon-bearing gases to calculate the ratio of air to fuel.

We have found that providing a uniform method to relate the specific exhaust gas constituents to air/fuel balance (independent of the quality of the combustion process or the power produced) makes the engine tuner's job much easier – and easier to understand as well.

It is important to actually use the Lambda value as calculated above in practice to see how well it correlates to the real world. A little experience goes a long way in building confidence as to the efficacy of this parameter.

The Relationship between Lambda and Equivalence Ratio:

Equivalence Ratio is also used to evaluate the balance between oxygen available and oxygen required for complete combustion. Equivalence Ratio is simply the ratio of oxygen required to oxygen available, so it is effectively an oxygen demand calculation, and is the inverse of Lambda.

$$\text{ER} = 1 / \text{Lambda} \quad \text{Thus, if Lambda is 0.950, ER would be } 1/0.950 = 1.053$$

The effect of NO_x on Lambda:

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NO has a relatively immaterial effect on the lambda calculation, as 1,000 ppm NO is only equivalent to 0.05% Oxygen (O₂) utilization, yielding an error of 0.002 in the lambda calculation if it is ignored. Thus, a 4-gas EGA is adequate for accurate lambda calculation - but at least 4 gases **must** be measured.

The effect of Oxygenated fuels on Lambda:

Oxygenated fuels contain a very small amount of oxygen in the fuel, which is released as the fuel is burned. The total O₂ equivalence in typical oxygenated fuel is on the order of 0.10% O₂, so this effect is also very small.

The effect of various ‘octane’ fuel mixes on Lambda:

Various mixes of gasoline contain differing ratios of short and long hydrocarbon chains, resulting in a variation of octane rated fuels. This has a small effect on the ratio of hydrogen to carbon in the fuel, but these variations have a trivial effect on the lambda calculation.

Sample Dilution and Air Injection Effects on Lambda:

As a side note, it is important to understand the effect that sampling air leaks or outright air injection may have on lambda calculation. **The percentage of extra air in the exhaust gases will result in the same percentage error in the Lambda calculation.**

I.E, a 5% air leak will not only dilute (lower) the CO, HC, CO₂ and NO_x gas readings by 5%, but will increase the Oxygen reading by about 1.00% (5.0% of 20.9%) and will result in the calculated Lambda being 5% leaner than it should. That means that a perfect Lambda of 1.000 will be reported as 1.050 if there is 5% air leak or injection.

This is a significant error, and can occur relatively easily. It should be noted that air leaks or injection will always bias the lambda calculation toward the lean side – so they should be dealt with and corrected before any lambda calculations using measured gases are attempted.

Air injection should be disabled for Lambda to be calculated correctly.

The effect of Combustion Efficiency (and misfires) on Lambda:

Because the Lambda calculation determines the balance between Oxygen and combustible gases by comparing all the oxygen available to the combustibles bearing gases – it is relatively insensitive to the degree to which the combustibles have been oxidized. Thus, an engine misfire has absolutely no effect on the balance calculation.

In essence, because all of the gases are used in the lambda calculation, the gas mix in the intake manifold, half-way through the combustion process, before a catalytic converter,

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of at the tailpipe will ALL yield the same Lambda result. The intake manifold will contain Oxygen, HC, and no CO, CO₂, or NO_x. They will, however be in balance. The tailpipe should contain low levels of Oxygen and HC and CO (the sources of combustion), but high levels of CO₂ and water vapor. They will be at the same balance as the intake manifold gases. It really does not matter where the gases are measured, or how efficient the combustion process is operating.

Pre and Post CAT gases – the effect of CAT Induced Oxidation on Lambda:

Similar to the engine misfire situation above, the CAT converter simply increases the combustion efficiency of the overall fuel oxidation process. Therefore, the engine out gases, which probably are no better than 95% oxidized should calculate the same lambda value as the CAT out gases, which have been oxidized to a CE of 99% or higher. The gas stream after the CAT should calculate at the same Lambda value as the gases before the CAT.

This ability to calculate Lambda independent of Combustion Efficiency is a very valuable feature of the Brettschneider equation – as fuel management control may be verified independent of other mitigating factors during engine diagnostics by this method.

Using Lambda for Performance Tuning:

Performance-tuning generally prioritizes power above efficiency. This is generally accomplished by both maximizing the induction charge volume and making sure that as much of the oxygen as possible is used up in the oxidation process. Due to these two criteria, performance tuners generally tune to the rich side of stoichiometric (Lambda less than 1.000). This ensures that the cylinder which runs the most lean under the worst-case scenario will always have enough fuel to completely consume all the available oxygen – thereby producing the maximum power.

Due to this situation, the target Lambda value desired is engine and application-specific. The tuner should tune for the desired effect, and then determine the resulting Lambda obtained so this value can be used for later tuning confirmation and to replicate the tuning process.

In the past, Carbon Monoxide has been used as an indicator of Lambda – as generally the CO level increases fairly linearly with decreasing Lambda. As an example, a Lambda of 0.900 (10% rich – and A/F ratio of 13.25 for gasoline) yields about 3.3% Carbon Monoxide – providing the engine is operating at high Combustion Efficiency. This is, in fact, the downside of using CO alone as an indicator of Air/Fuel balance – as the concentration of CO in the exhaust gas is a function of both the oxygen/combustibles balance and the

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operating efficiency of the engine. Lambda is not, so it is a superior measure of air/fuel balance.

This ability to calculate Lambda independent of Combustion Efficiency is a very valuable feature of the Brettschneider equation – as fuel management control may be verified independent of other operating factors during engine diagnostics and tuning by using this parameter.

Also, see CE – Combustion Efficiency – for more information on the separate and mutual use of these higher-level parameters.

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