White Paper

Title: Lambda Calculation – The Brettschneider Equation, general principles and methods, and its use with alternate fuels
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Created by: R. Schrader
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The Brettschneider equation is the de-facto standard method used to calculate the normalized air/fuel balance (Lambda) for domestic and international I&M inspection programs. It is taken from a paper written by Dr. Johannes Brettschneider, at Robert Bosch in 1979 and published in "Bosch Technische Berichte", Vol 6 (1979) N0. 4, Pgs 177-186. In the paper, Dr. Brettschneider established a method to calculate Lambda (Balance of Oxygen to Fuel) by comparing the ratio of oxygen molecules to carbon and hydrogen molecules in the exhaust. The equation is a little complex, but is relatively easily calculated from the measured values of CO, CO2, unburned HC, and unconsumed

$$\lambda = \frac{[CO_2] + \left[\dfrac{CO}{2}\right] + [O_2] + \left[\dfrac{NO}{2}\right] + \left(\left(\dfrac{H_{cv}}{4} \times \dfrac{3.5}{3.5 + \dfrac{[CO]}{[CO_2]}}\right) - \dfrac{O_{cv}}{2}\right) \times ([CO_2] + [CO])}{\left(1 + \dfrac{H_{cv}}{4} - \dfrac{O_{cv}}{2}\right) \times ([CO_2] + [CO] + (n \times [HC]))}$$

Where :

$[XX]$ = Gas Concentration in % Volume.

H_{cv} = Atomic ratio of Hydrogen to Carbon in the fuel.

O_{cv} = Atomic ratio of Oxygen to Carbon in the fuel.

n = Number of Carbon atoms in each of the HC molecules being measured.

 n = 6 for Hexane (Gasoline), 3 for Propane (LPG), 1 for Methane (CNG)

O2 in the exhaust:

The equation above compares all of the oxygen in the numerator, and all of the sources of carbon and hydrogen in the denominator. (Water concentration is determined by as a

Bridge Analyzers, Inc.   (510) 337-1605 office
PO Box 39325   (216) 274-9262 fax
Solon, Ohio 44139   USA   support@bridgeanalyzers.com

fraction of the sum of CO2 and CO, and the ratio of CO to CO2 by the '3.5' term in the numerator). The result of the Brettschneider equation is the term 'Lambda' (λ) a dimensionless term that relates nicely to the stoichiometric value of air to fuel. At the stoichiometric point, Lambda = 1.000. A Lambda value of 1.050 is 5.0% lean, and a Lambda value of 0.950 is 5.0% rich. Once Lambda is calculated, A/F ratio can be easily determined by simply multiplying Lambda times the stoichiometric A/F ratio for the fuel used - e.g. 14.71 for gasoline.

Details of the Brettschneider Equation:

Although this equation may be difficult to understand in theory, it is simple to use in practice. The equation directly reflects the 'degree of lean-ness' of the air/fuel mixture – and is largely independent how efficiently the fuel is oxidized – a very important factor to consider when dealing specifically with air / fuel balance issues. The manner in which this equation is to be used is strictly a function of the application though, and it is an excellent replacement for more commonly used conventions, such as CO measurement for rich-side applications (performance tuning), 'wide range lambda sensors', which are not only very non-linear, but also very sensitive to combustibles in the exhaust stream, or EGT, which is a combination of flame temperature and volume (power).

The only stable air/fuel ratio measurement that we have found to date is one that first makes an accurate measure of the constituent gases in the exhaust stream (at least the four gases of HC, CO, CO2 and O2) and calculates the oxygen and combustibles content and then the lambda and A/F value as above.

The Relationship between Lambda and A/F ratio:

Because Lambda = 1.000 when the oxygen and combustibles are in perfect stoichiometric balance, Lambda can easily be used to calculate A/F ratio for particular fuels.

The active A/F ratio is simply the calculated Lambda times the stoichiometric A/F ratio for the specific fuel used (14.71 for gasoline, but other fuels have different values – see below) This method is far superior to other approaches which use only one gas (CO or Oxygen) to approximate A/F ratio – as the Brettschneider method uses all of the oxygen and carbon-bearing gases to calculate the ratio of air to fuel.

We have found that providing a uniform method to relate the specific exhaust gas constituents to air/fuel balance (independent of the quality of the combustion process or the power produced) makes the engine tuner's job much easier – and easier to understand as well.

It is important to actually use the Lambda value as calculated above in practice to see how well it correlates to the real world. A little experience goes a long way in building confidence as to the efficacy of this parameter.

The effect of NOx on Lambda:

NO has a relatively immaterial effect on the lambda calculation, as 1,000 ppm NO is only equivalent to 0.05% Oxygen utilization. A 4-gas analyzer is adequate for lambda calculation - **but at least 4 gases must be measured.**

Sample Dilution and Air Injection Effects on Lambda:

As a side note, it is important to understand the effect that sampling air leaks or outright air injection may have on lambda calculation. **The percentage of extra air in the exhaust gases will result in the same percentage error in the Lambda calculation.**

I.E, a 5% air leak will not only dilute (lower) the CO, HC, CO2 and NOx gas readings by 5%, but will increase the Oxygen reading by about 1.00% (5% of 20.9%) and will result in the calculated Lambda being 5% leaner than it should. That means that a perfect Lambda of 1.000 will be reported as 1.050 if there is 5% air leak or injection. This is a significant error, and can occur relatively easily. It should be noted that air leaks or injection will always bias the lambda calculation toward the lean side – so they should be dealt with and corrected before any lambda calculations using measured gases are attempted.

Air injection should be disabled for Lambda to be calculated correctly.

The effect of Combustion Efficiency on Lambda:

Because the Brettschneider equation calculates the balance between Oxygen and Combustibles by comparing all the oxygen and combustibles in all the gases, independently of how well they are oxidized, it is relatively insensitive to the degree to which the combustibles have been burned – referred to as Combustion Efficiency. Thus, the lambda and A/F readings on the analyzer are accurate no matter how well the engine is operating – or if you look at the gases pre-CAT or post-CAT, or even in the intake manifold! Since all the oxygen and combustibles atoms are being accounted for, the balance between them can be calculated no matter how well they are burned.

This ability to calculate Lambda independent of Combustion Efficiency is a very valuable feature of the Brettschneider equation – as fuel management control may be verified independent of other mitigating factors during engine diagnostics by this method.

Pre and Post CAT gases –

As explained above, as Lambda is a balance equation, it does not matter if the combustibles are well oxidized or not – so it does not matter if pre or post CAT gases are used – Lambda will calculate to the same value with gases from either location.

Alternative Fuels Considerations

Lambda and A/F Ratio factors as a function of Fuel:

Various fuels have unique, fuel-dependant factors used to calculate Lambda and A/F Ratio. The Lambda calculation factors are significantly different for gasoline, LPG and CNG and even vary by the gasoline blend, (and are actually different from season to season). While the differences in gasoline blends are relatively small, the differences between gasoline and LPG and CNG are greater.

An example of the variation of important fuel constants is given below:

Fuel Parameters – their values and effect on Lambda and A/F Ratio Calculation:

Fuel Factor	American (BAR) Oxygenated Gasoline (Hexane)	Propane (LPG)	Methane (CNG)
Hcv	1.9800	2.667	4.000
Ocv	0.0170	0.000	0.000
Cfactor (n)	6.000	3.000	1.000
A/F Nom	14.71	15.87	17.45

NOTE: Bridge Exhaust Gas analyzers use constants above for the fuel selected when calculating Lambda, A/F Ratio, and Combustion Efficiency, so the value displayed by the analyzer is correct for the fuel being used.